A DEEPER LOOK AT NONDEGENERACY IN ONE DIMENSION

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- All the second order differential equations admit two linear independent solution.

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- Then why Schrȯdinger equation for bound state in one dimension admit only one solution??

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- What's about the other solution?
  Our intuition tells that the other solution is somehow physically unacceptable.It might blow up.Lets take a look.

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- Now Wronskian is defined to be:

$$\triangle_{[\psi_1,\psi_2]} = \psi_1\frac{d\psi_2}{dx} - \psi_2\frac{d\psi_1}{dx}$$

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$$\frac{d\frac{\psi_2}{\psi_1}}{dx} = \frac{\triangle}{\psi_1^2} \implies \psi_2 = \psi_1 \int dx \frac{\triangle}{\psi_1^2} \tag{3}$$

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- Let ψ_1 be the **GOOD** behaved function.
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  *BINGO!!!!*We are done.

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So we have proved whatever be the potential in one dimension we will always have a solution blowing up at $\pm\infty$
We can not accept this solution.
This would remove the apparent discrepancy between Math and Physics!!!

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